Assignment 9

Coverage: 16.5, 16.6 in Text.

Exercises: 16.5 no 4, 8, 10, 13, 17, 19, 24, 33, 42, 48, 56; 16.6 no 4, 7, 10, 15.

Hand in 16.5 no 33, 48, 56; 16.6 no 10 by April 8.

Supplementary Problems

1. The zeros of a function $F(x, y, z) = 0$ may define a surface in space. Let $S = \{(x, y, z) : F(x, y, z) = 0\}$ where F is C^1. Suppose that $F_z \neq 0$. By Implicit Function Theorem the set S can be locally described as the graph of a function $z = \varphi(x, y)$. Suppose now $S = \{(x, y, \varphi(x, y)),\ (x, y) \in D\}$ where D is a region in the xy-plane. Derive the following surface area for S:
$$|S| = \iint_D \frac{|\nabla F|}{|F_z|}\, dA(x, y) \ .$$

2. Let $(x(t), y(t)),\ t \in [a, b]$, be a curve C parametrized by t in the first and the second quadrants. Rotate it around the x-axis to get a surface of revolution S.

   (a) Show that a parametrization of S is given by $(\alpha, t) \mapsto (x(t), y(t)\cos\alpha, y(t)\sin\alpha)$ $\alpha \in [0, 2\pi]$, and it is regular when C is regular.

   (b) Show that the surface area of S is given by
$$2\pi \int_C y(t)\, ds \ .$$

   (c) When $y = \varphi(x), x \in [a, b]$, where φ is C^1, the surface area becomes
$$2\pi \int_a^b \varphi(x)\sqrt{1 + \varphi'^2(x)}\, dx \ .$$